

3-D Rotations

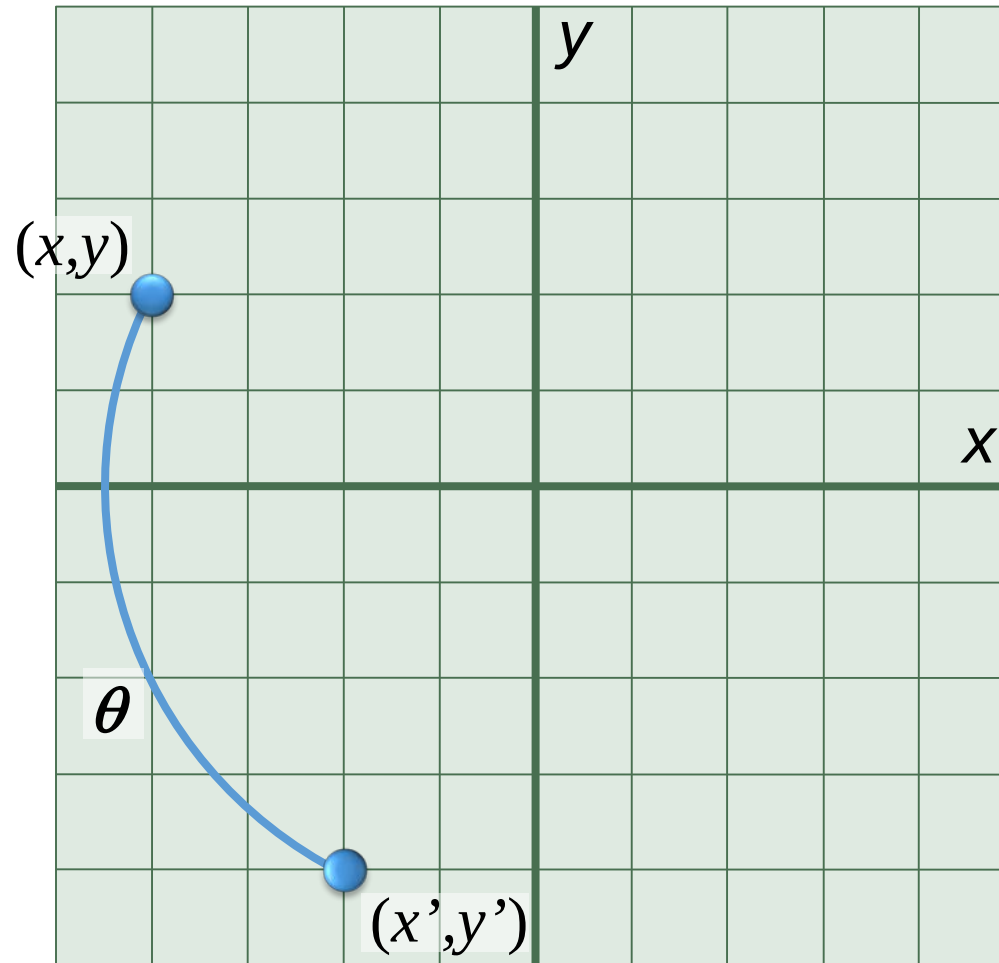
CS418 Computer Graphics

John C. Hart

3-D Rotations

- Recall 2-D rotation

$$\begin{bmatrix} \cos \theta & -\sin \theta & 1 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$



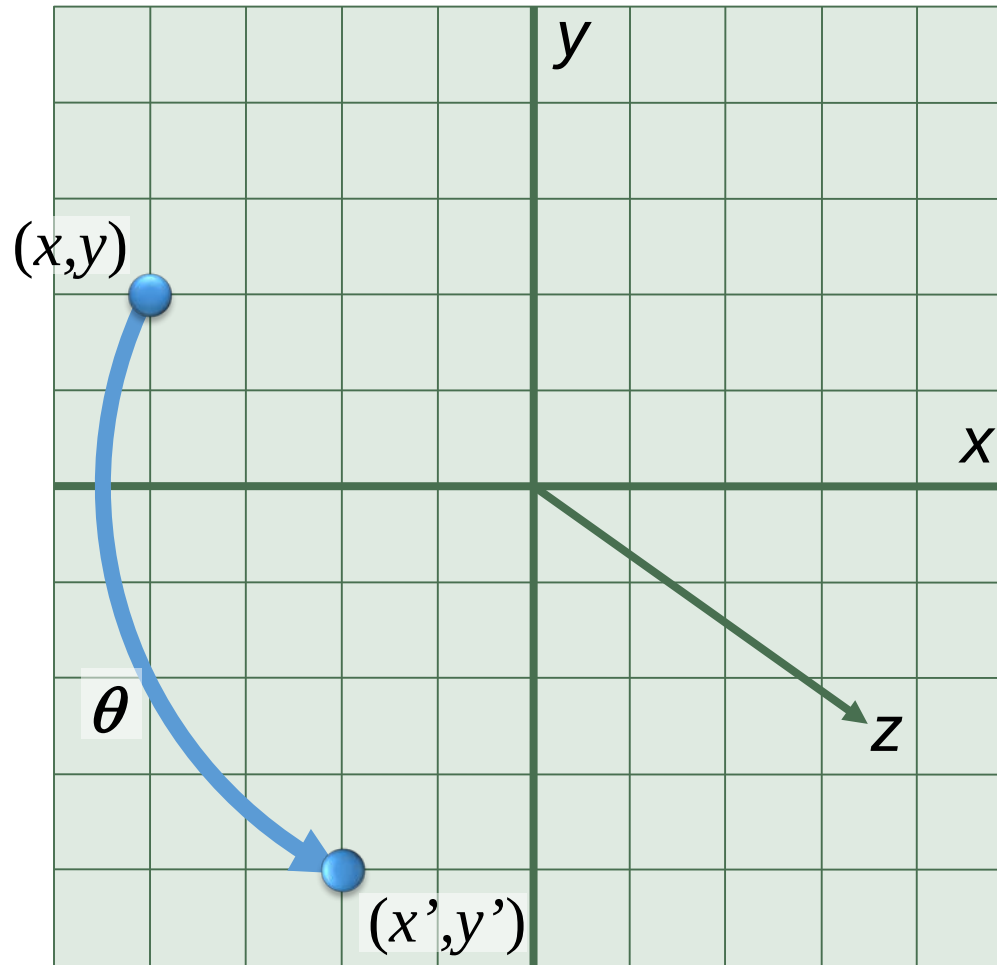
3-D Rotations

- Recall 2-D rotation

$$\begin{bmatrix} \cos \theta & -\sin \theta & \\ \sin \theta & \cos \theta & \\ & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

- Rotation about z-axis

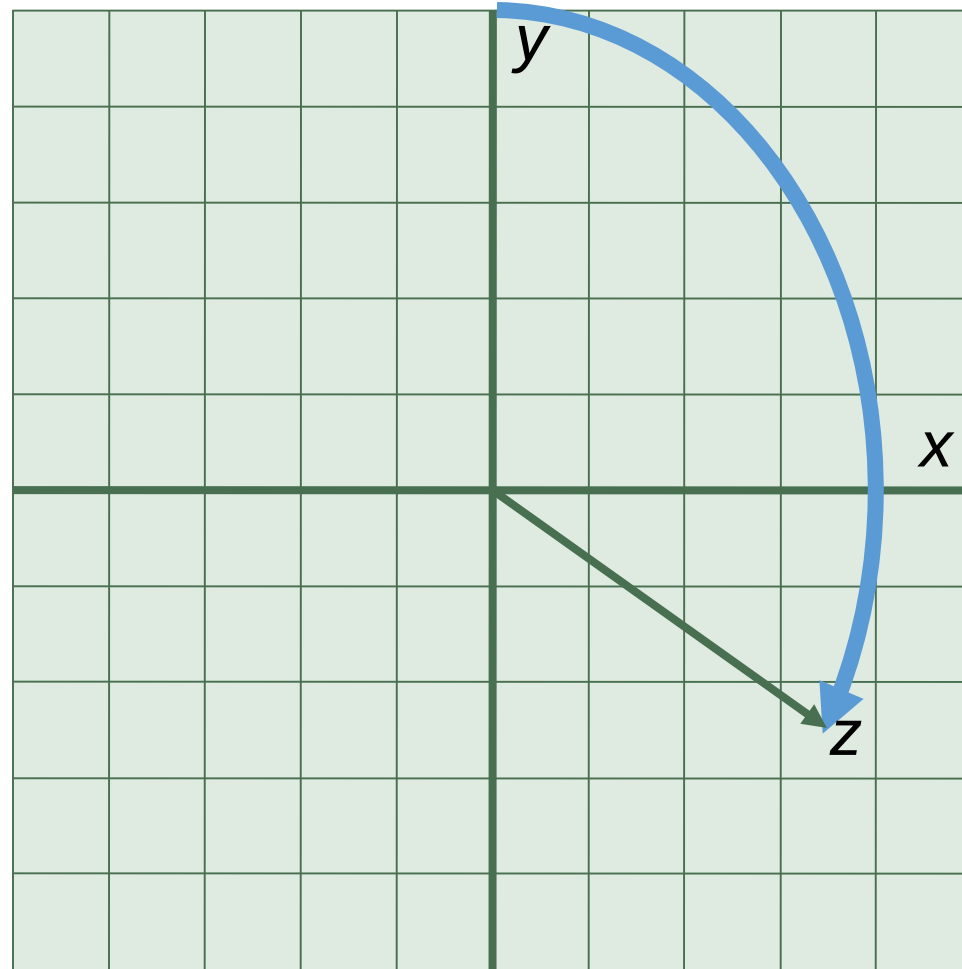
$$\begin{bmatrix} \cos \theta & -\sin \theta & & \\ \sin \theta & \cos \theta & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z \\ 1 \end{bmatrix}$$



3-D Rotations

- Rotation about x-axis

$$\begin{bmatrix} 1 & & \\ & \cos \theta & -\sin \theta \\ & \sin \theta & \cos \theta \\ & & & 1 \end{bmatrix}$$



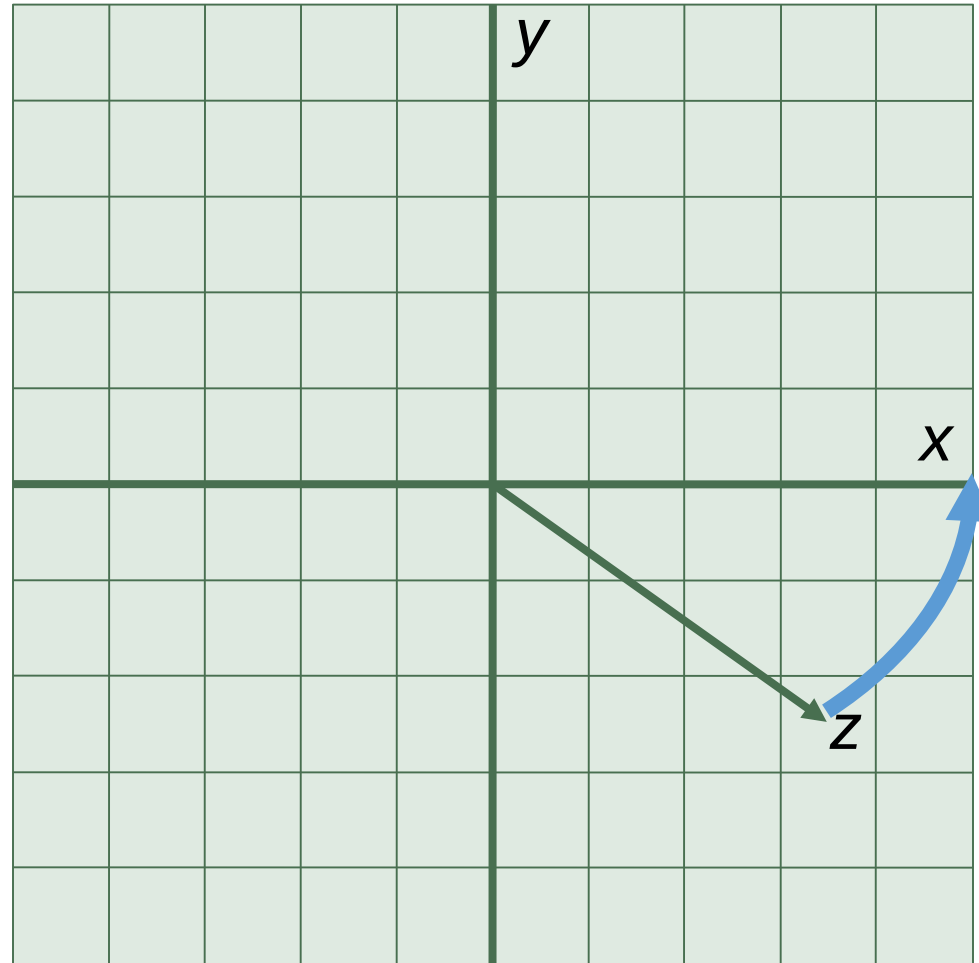
3-D Rotations

- Rotation about x-axis

$$\begin{bmatrix} 1 & & & \\ & \cos \theta & -\sin \theta & \\ & \sin \theta & \cos \theta & \\ & & & 1 \end{bmatrix}$$

- Rotation about y-axis

$$\begin{bmatrix} \cos \theta & & \sin \theta & \\ & 1 & & \\ -\sin \theta & & \cos \theta & \\ & & & 1 \end{bmatrix}$$



Some Rotation Fun Facts

- A rotation matrix is orthogonal
 - Columns are mutually orthogonal unit vectors
 - Rows are mutually orthogonal unit vectors
- The inverse of a rotation matrix equals its transpose
- The product of two rotation matrices is a rotation matrix

Try these...

$$\begin{bmatrix} 1 & & & \\ & \cos \theta & -\sin \theta & \\ & \sin \theta & \cos \theta & \\ & & & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos \theta & & & \\ & 1 & & \\ & & \sin \theta & \\ -\sin \theta & & \cos \theta & \\ & & & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos \theta & -\sin \theta & & \\ \sin \theta & \cos \theta & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$$